_____

Image Coding based Orthogonal Polynomials Multiresolution Analysis with Joint Probability Context Modeling and Modified Golomb-Rice Entropy Coding

Puli Vinay Kumar[1] , Kurapati Veeranjaneya Vara Prasad[2], Dr.T.Srikanth[3] , Ms. Vadla Anuja[4] Dr. Deena Babu Mandru[5]

Puli Vinay Kumar[1]
Assistant professor,Department of IT,
Malla Reddy Engineering College(A),
Maisammaguda, Medchal-Malkajgiri, Telangana State - 500100, India

Kurapati Veeranjaneya Vara Prasad[2]
Assistant professor,Department of CSE,
Malla Reddy Engineering College(A),
Maisammaguda, Medchal-Malkajgiri, Telangana State - 500100, India

Dr.T.Srikanth[3]
Associate Professor, Department of CSE,
Malla Reddy Institute of Technology and Science(A),
Maisammaguda, Medchal-Malkajgiri, Telangana State - 500100, India

Ms. Vadla Anuja[4]
Assistant Professor, Department of CSE-IoT,
Malla Reddy Engineering College(A),
Maisammaguda, Medchal-Malkajgiri, Telangana State-500100, India

Dr. Deena Babu Mandru[5]
Assistant professor,Department of IT,
Malla Reddy Engineering College(A),
Maisammaguda, Medchal-Malkajgiri, Telangana State - 500100, India

Abstract —The work proposes, a JPEG2000 like compression technique which is based on multiresolution analysis of orthogonal polynomials transformation (OPT) coefficients has been presented with bit modeling for Golomb-Rice entropy coding. Initially, the image under analysis is divided into blocks and OPT is applied to each divided blocks. Then, transformed coefficients are represented as sub bands like structure (multiresolution) and scalar quantization is carried out to the transformed coefficients to reduce the precision. The quantized coefficients are then bit modelled in the bit plane using a joint probability statistical model, and significant bits in the bit plane are chosen. On the selected relevant bits, a geometrically distributed set of context is modelled for further encoding with modified Golomb-Rice encoding to provide compressed data. The decompression procedure is just the reverse of compression procedure. Experiments and analysis are carried out to demonstrate the efficiency of the proposed compression scheme in terms of compression ratio and Peak-Signal-to Noise Ratio (PSNR), and the results are encouraging

Keywords: Bit Modeling, Golomb-Rice Coding, Image Compression, Multiresolution Analysis and Orthogonal Polynomials.

I. Introduction

In recent years, multimedia data plays a vital role in transmission systems. With latest developments in digital camera, images are captured, stored and transmitted vastly and hence requires huge amount of storage, making the transmission inconvenient. To solve this problem, many compression schemes are developed. In general, Compression methods [1] can be classified into two categories: lossy and lossless. In lossy compression, the data size is reduced for efficient storage with higher compression ratio (CR) and used in areas such as streaming media and Internet telephony. In contrast, lossless compression scheme reconstructs the original data perfectly from compressed data and finds use in fields such as medical and satellite images. This leads to the development of two popular compression standards, Joint Photographic Expert Group (JPEG) and JPEG2000.

JPEG [2] uses Discrete Cosine Transform (DCT) and gives good compression ratio for natural imagery. However, it produces blocking artefact when higher compression is performed. JPEG2000 [3], a recent still-image compression method is based on wavelet technique and provides good objective and subjective image quality than JPEG methods at higher compression ratio. Also, the JPEG2000 gives better compression ratio, Region of Interest (ROI) coding, quality scalability and resolution scalability than JPEG standard. But, the main weakness is the occurrence of ringing artifacts at higher compression ratios. Different variants to JPEG2000 are suggested to improve the quality of reconstructed image at low bit rates. One such JPEG2000 like compression scheme is

2587

_____

investigated in this paper, with orthogonal polynomials based multiresolution analysis of transform coefficients, and a new bit modeling that can best suite Golomb-Rice coding.

II.   Literature Survey

This section deals with literature survey related to the proposed image compression scheme with emphasis on different variants of JPEG2000 is presented. Yair Wiseman [11] reported a non-suitability nature of JPEG scheme for Global Positioning System (GPS) images and also designed an improved JPEG compression scheme with suitability to GPS images. Rong Zhang et al. [12] reported a bit plane entropy coder based on Laplacian distribution of the wavelet coefficients. Since, Embedded Block Coding with Optimized Truncation (EBCOT) used in JPEG2000, consumed most of the processing time due to bit operations, a pipelined Binary Arithmetic Coder (BAC) was designed in [13] that could encode the bits in higher rates than traditional BAC. [14] gives a lossless image compression and decompression scheme where Differential-Differential Pulse Coded Modulation (DDPCM) to reduce both bandwidth and parallel Golomb-Rice coding were utilized. Chang-Hoon Son et al. [15] designed a low-complexity embedded-compression (EC) algorithm for JPEG2000 encoder system and claimed to reduce memory requirements by a factor of 2 than direct implementation of JPEG2000 encoder.

Plane fitting with inter block prediction based image compression scheme was reported in [16] wherein higher compression ratio is achievable at the expense of quality degradations. Chrysafis and Antonio Ortega [17] reported an image coding algorithm based on backward-adaptive quantization of classification technique simultaneously focusing the image context for entropy coding in wavelet domain. Similar to [17], Zhen Liu and Lina J. Karam [18] analysed contexts used in JPEG2000 that was closely related to compression performance. The authors stated that contexts in JPEG2000 capture the correlations in wavelet coefficients very well and also claimed that their method could produce similar coding performance as JPEG2000 with smaller number of coefficients. Bruno Aiazi et al. [39] suggested a lossless image compression scheme based on linear-regression prediction with fuzzy techniques. Entropy based image compression algorithm was reported in [35] where it reduced the number of coding parameters by a factor of about 10 compared with JPEG standard.

In [19], David Taubman reported a compression algorithm based on independent Embedded Block Coding with Optimized Truncation of the embedded bit streams. The author claimed that the compression algorithm could achieve modest complexity and suitable for remote browsing of large compressed images. Multiresolution human visual system and statistically based image coding scheme was reported in [20] where the lifting based wavelet transform was used to decorrelate the input image. Henrique S. Malvar [21] designed an entropy coder that combined Run-length and Golomb-Rice encoders and could automatically switch between the two modes based on simple rules, adjusting the encoding parameters with previous output codeword. Jianhua Chen [22] directly coded the significance map symbols and sign symbols

of embedded wavelet coding scheme with designed context models.

A Discrete Tchebichef Transform (DTT) based image compression algorithm was designed by Ranjan K. Senapati et al. [33, 41] and reported that the properties of DTT were similar to DCT with energy compactness. Similar to [33 and 41], Paulo A. M. Oliveira et al. [34] reported a DTT approximation for image and video coding. The forward and inverse DTT were multiplication–free and required a minimal number of additions and bit-shifting operations, besides significantly low arithmetic cost. Lossless colour image compression algorithm based on context-adaptive arithmetic coding and hierarchical prediction was reported in [36]. In this approach, to achieve lossy compression, the original image was first decorrelated by reversible colour transform and then the Y component was encoded by conventional grey scale image compression method. Subsequently, hierarchical scheme was used for chrominance images that could reduce the number of bit than JPEG2000 and JPEG-XR methods. An algorithm to optimize Huffman coding, run-length coding, quantization table on par with JPEG standard was designed in [37] and achieved a PSNR gain of up to 3 dB compared to baseline JPEG. Similar to [37], a graph-based algorithm is reported in [38] to jointly optimize Huffman coding ,Run-length coding and Quantization table. Computationally efficient encoders and decoders for lossy compression with sparse regression code were reported in [40].

Independently, use of multiresolution image analysis to study different image processing is reported in the literature. Minh N. Do and Martin Vetterli [7] introduced a scheme to capture geometrical structure for visual information with contourlet transform as multiresolution representation. John Michael Lounsbery [8] made use of multiresolution analysis to decompose complicated function to simpler low resolution part for analysing surfaces of arbitrary topological type. Utilization of multiresolution analysis for image retrieval is reported in [29]. Babb et al. [6] reported a multiresolution analysis transform for image compression in a constrained quantization approach. A near lossless image compression with multiresolution analysis of wavelet is reported in [4, 5].

It is clear from the literature that efficient representation of visual information, such as multiresolution analysis plays a significant role in image compression, besides a need of proper bit modeling of transform coefficients that can capture the contexts for utilization in entropy coding. Hence in this paper a multiresolution representation of orthogonal polynomials coefficients, followed by a geometrical modeling of bit plane correlation with probability for use with Golomb-Rice Coding is proposed to device a new JPEG2000 like image compression technique. The choice of orthogonal polynomials coefficients [24] in this paper is two folded: The conventional DCT, used in JPEG is poor at approximating discontinuities or impulse in imagery signal, besides higher computational complexity [9]. Similarly DWT, used in JPEG2000, could not well suited to represent 2-D singularities along edges or contours [10]. In the other fold, orthogonal polynomials coefficients are found to have excellent sparse representation besides energy preservation in transformed domain with multiresolution representation [29].

_____

The effectiveness of such an orthogonal polynomials multiresolution analysis to devise geometrically distributed set of contexts for entropy coding with Golomb-Rice coding is also investigated.

The rest of the paper is organized as follows. Section 3 briefly introduces the mathematical preliminaries on orthogonal polynomials model and its basis operators. Section 4 gives the architecture of the proposed image compression algorithm is presented. The proposed image compression technique is presented in detail in section 5. The performance measures are described in Section 6. Section 7 gives the experiments, results and discussions carried out to show the effectiveness of the proposed system. and finally conclusions are drawn in Section 8.

III. Mathematical Preliminary On Orthogonal Polynomials

The orthogonal polynomials transformation that has already been established to extract edge [23, 42], image coding [24, 25, 30], image retrieval [26-29], image security [43] and texture enhancement [32] is utilized in this proposed work to design a new compression technique. This section provides a quick overview of the subject. To propose the image compression scheme, a linear 2-D image formation system based on a Cartesian coordinate separable, blurring, point spread operator is considered, in which the image I results in the superposition of the point source of the impulse weighted by the value of the object function f. It is particularly beneficial for image analysis to express the object function f in terms of derivatives of the image function I relative to its Cartesian coordinates. The point-spread function $M(x, y)$ can be considered as a real valued function defined for $(x, y) \in X \times Y$, where X and Y are ordered subsets of real values. In the case of grey-level image of size $(n \times n)$ where X (rows) consists of a finite set, which be labelled {0, 1, …, n-1} for convenience, the function $M(x, y)$ reduces to a sequence of functions.

$$M(i, t) = u_i(t), \quad i, t = 0, 1, …, n-1 \tag{1}$$

The linear 2-D transformation can be defined using point-spread operator $M(x,y)$ ($M(i, t) = u_i(t)$) as shown in equation (2).

$$\beta'(\zeta, \eta) = \int_{x \in X} \int_{y \in Y} M(\zeta, x) M(\eta, y) I(x, y) dx dy \tag{2}$$

Considering both X and Y to be finite sets of values {0, 1, 2, …, n −1}, equation (2) can be written in matrix notation as follows:

$$\left| \beta'_{ij} \right| = \left(|M| \otimes |M| \right)^t |I| \tag{3}$$

where $\otimes$ is the outer product, $\left| \beta'_{ij} \right|$ are n^2 matrices arranged

in dictionary sequence, $|I|$ is the image, $\left| \beta'_{ij} \right|$ are the coefficients of transformation and the point spread operator $|M|$ is

$$|M| = \begin{vmatrix} u_0(x_0) & u_1(x_0) & \mathrm{L} & u_{n-1}(x_0) \\ u_0(x_1) & u_1(x_1) & \mathrm{L} & u_{n-1}(x_1) \\ & & \mathrm{M} & \\ u_0(x_{n-1}) & u_1(x_{n-1}) & \mathrm{L} & u_{n-1}(x_{n-1}) \end{vmatrix} \tag{4}$$

A set of orthogonal polynomials $u_0(t)$, $u_1(t)$, …, $u_{n-1}(t)$ of degrees 0, 1, 2, …, n-1, respectively are considered, to construct polynomial operators of varying sizes from equation (4) for $n \geq 2$ and $t_i = i$. The generating formula for the polynomials is as follows:

$$u_{i+1}(t) = (t - \mu) u_i(t) - b_i(n) u_{i-1}(t) \text{ for } i \geq 1,$$
$$u_1(t) = t - \mu, \text{ and } u_0(t) = 1,$$

where $b_i(n) = \dfrac{\langle u_i, u_i \rangle}{\langle u_{i-1}, u_{i-1} \rangle} = \dfrac{\sum_{t=1}^{n} u_i^2(t)}{\sum_{t=1}^{n} u_{i-1}^2(t)}$ and $\mu = \dfrac{1}{n} \sum_{t=1}^{n} t$

Considering the range of values of t to be $t_i = i$, $i = 1, 2, 3$ …n, we obtain

$$b_i(n) = \frac{i^2(n^2 - i^2)}{4(4i^2 - 1)}, \qquad \mu = \frac{1}{n} \sum_{t=1}^{n} t = \frac{n+1}{2} \tag{5}$$

The point-spread operators $|M|$ of different size can be constructed from equation (4) using the above orthogonal polynomials for $n \geq 2$ and $t_i = i$. The elements of $|M|$ are scaled to integer values, to make point-spread operations more convenient. [24, 25, 29, 31] gives the extensive explanation of OPT operators can be obtained. The basis operators required for the purpose of inverse operations are presented in the next subsection.

A. Orthogonal Polynomials Basis

For the sake of computational simplicity, the finite Cartesian coordinate set X, Y is labelled as {1, 2, 3}. The point spread operator in equation (3) that defines the linear orthogonal transformation for image coding can be obtained as $|M| \otimes |M|$, where $|M|$ can be computed and scaled from

$$\|M\| = \begin{vmatrix} u_0(x_0) & u_1(x_0) & u_2(x_0) \\ u_0(x_1) & u_1(x_1) & u_2(x_1) \\ u_0(x_2) & u_1(x_2) & u_2(x_2) \end{vmatrix} = \begin{vmatrix} 1 & -1 & 1 \\ 1 & 0 & -2 \\ 1 & 1 & 1 \end{vmatrix} \tag{6}$$

The polynomial basis operators set O_{ij}^n ($0 \leq i, j \leq n-1$) can be computed as $O_{ij}^n = \hat{u}_i \otimes \hat{u}_j$ where $\hat{u}_i$ is the $(i + 1)^{st}$ column vector of $|M|$.

2589

_____

IV PROPOSED IMAGE COMPRESSION SCHEME

In this section utilization **of** orthogonal polynomials transformation described in the previous section, is highlighted to propose an orthogonal polynomial multiresolution analysis based image compression technique. The architecture of the proposed compression technique is presented in the next section.

A.   ARCHITECTURE

The proposed image compression technique initially divides the image under analysis into blocks and each block is subjected to orthogonal polynomials transformation as described in section 3. Then, the transformed coefficients are represented in multiresolution form and subsequently, the coefficients are scalar quantized for further coding. The quantized coefficients are represented in bit planes to select significant bits which are carried out with a selection of MSB and modeling the bits with joint probability of neighbouring ones. After selecting the significant bits from all the sub bands, the proposed technique models it as geometrically distributed set of context for further entropy encoding. The proposed compression technique modifies the Golomb-Rice entropy coding for this purpose. In the proposed modified Golomb-Rice entropy coding, the Most Probable Bit (MPB) is calculated from all the significant bits of the sub
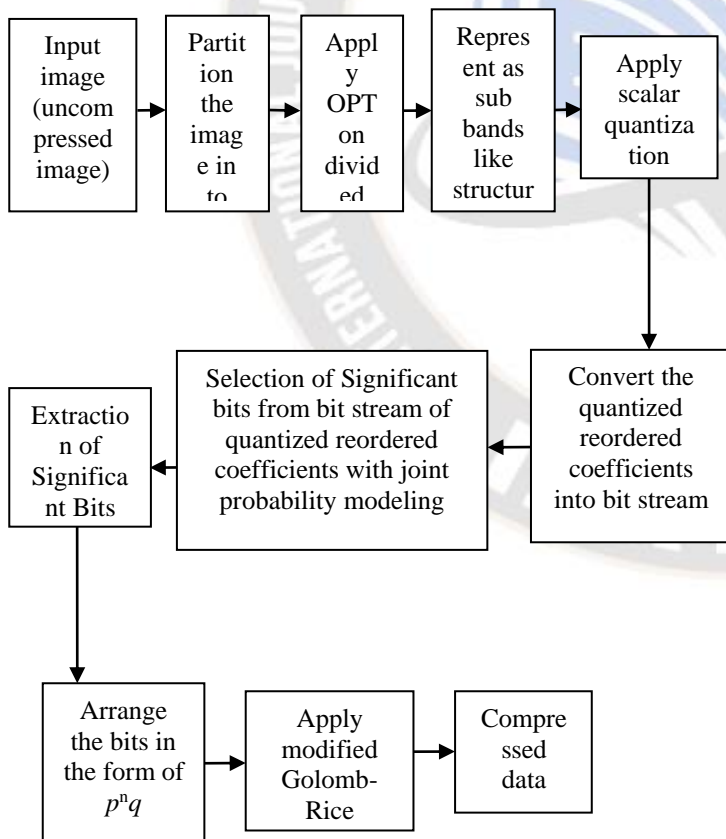

Figure 1. Architecture of Proposed Orthogonal Polynomials Based Multiresolution Analysis Image Compression Technique

bands and are used to derive parameters q and r of each bit plane. Finally, the parameters q and r are arranged as unary and binary encoding format to produce the compressed data. The architecture of the proposed orthogonal polynomials based multiresolution analysis image compression technique is presented in Figure 1.

B.   *Proposed Coding Technique*

In this sub section, the steps involved in the design of the proposed image compression technique is presented. The proposed work consists of the mentioned four major steps.
i)   Multiresolution reordering.
ii)   Scalar quantization.
iii)   Selection of Significant bits with joint probability modeling and,
iv)   Modified Golomb-Rice entropy coding.
A detailed description of each of these steps in the design of proposed compression technique is presented is subsequent subsections.
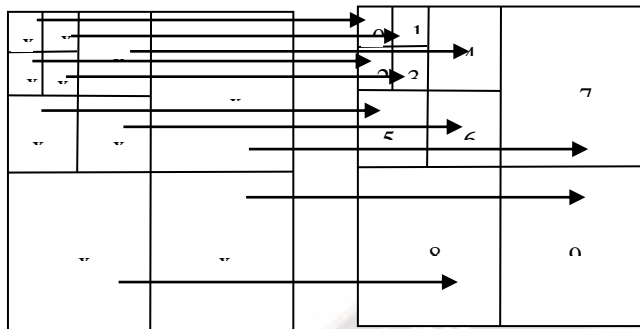
C.   *Multiresolution Reordering*

The orthogonal polynomials coefficients β_{ij}' obtained as described in section 3 are reordered to provide image sub bands in multiresolution decomposition like structure. The extensive explanation about multiresolution reordering of orthogonal polynomials coefficient utilized in this paper, can be found in [29]. The transformed coefficients β_{ij}' are reordered into $(3\log_2 N + 1)$ multi resolution sub bands for each $(N \times N)$ block where N is a power of 2. To reorder the transformed coefficients, following assumptions are made. For a coefficient β_{ij}', let $2^{a-1} \leq i \leq 2^a$ and $2^{b-1} \leq i \leq 2^b$ where a and b are integers and $i, j = 0,\ldots, (N\text{-}1)$. Now, β_{ij}' coefficient is arranged in to a particular sub band S_y with y computed as

$$y = \begin{cases} 0 & \text{for } m = 0 \\ 3(m-1) + 2\left(\dfrac{a}{m}\right) + \left(\dfrac{b}{m}\right) & \text{otherwise} \end{cases}$$

where $m = \max(a,b)$. The process is repeated for each block of the image under analysis with size $(R \times C)$ where R and C are height and width respectively. Then the reordered location representing various levels of the orthogonal polynomials transformed coefficient β_{ij}', is determined with the function presented as follows for the block $B(z,w)$ where z and w represent the row and column of the block B.

_____

$$R = (2^{m-1}z + i - 2^{a-1}, 2^{m-1}w + j - 2^{b-1})$$

The original orthogonal polynomials transform coefficients are depicted in Figure 2(a) and the corresponding multiresolution representation of subbands are presented in Figure 2(b).

(a)   **(b)**
Figure 2.Multiresolution Analysis (a) Transformed coefficients of (8x8) block (b) Reordered structure.

In this proposed coding scheme, the multiresolution representation of orthogonal polynomials transformed coefficients, thus obtained is subjected to scalar quantization, to reduce the precision and the same is presented in the next section.

D. *Scalar Quantization*

After multiresolution ordering of sub bands like structure, the transformed coefficients require more storage, and hence needs to be coded efficiently for transmission and storing. In the proposed image compression scheme, scalar quantization is performed on the orthogonal polynomials reordered coefficients to reduce the precision as described in JPEG2000. The formula for calculating quantization value β'_{ijq} on each coefficient β'_{ij} is as follows.

$$\beta'_{ijq} = \frac{\left\lfloor \left| \beta'_{ij} \right| \right\rfloor}{\Delta}$$

(7)

Where Δ the quantizer is step size and $\lfloor \ \rfloor$ is the floor

function. If the value of Δ is taken to be 1, then no quantization process is effective and gives birth a lossless scheme. In order to quantize the reordered orthogonal polynomials transformed coefficients, the quantizer step must be greater than 1. The scalar quantized orthogonal polynomials reordered coefficients thus obtained, are required to be effectively entropy coded so that better compression ratio is achievable. Hence in this proposed orthogonal polynomials based multiresolution analysis coding scheme, a new bit modeling of quantized coefficients is proposed and the same is presented in the next section.

E. *Selection of Significant Bits Based on Joint Probability*

The proposed orthogonal polynomials based multiresolution analysis image compression technique adaptively models the bit planes of scalar quantized coefficients based on the probability distribution of the total bits in the current bit plane. Therefore, the quantized orthogonal polynomials reordered coefficient is decomposed into N number of bit planes, where N depends on the image depth, ranging from N[th] bit plane (LSB) to 1st bit plane (MSB) that contain all the lower order and higher order bits. Decomposition of bit stream of orthogonal polynomials reordered coefficients $\beta'_{ijq}(x, y)$ into N number of bit planes is given in Equation (8).

$$\beta'_{ijq}(i, j) = \beta'_{ijq1}(i, j) + \beta'_{ijq2}(i, j) + \ldots + \beta'_{ijqN}(i, j)$$

(8)

where $\beta'_{ijq1}(i, j)$ to $\beta'_{ijqN}(i, j)$ are the N number of bit planes.

Next, to select the significant bits, the proposed orthogonal polynomials based multiresolution analysis image compression technique performs scanning column wise on the bit plane from $\beta'_{ijqN}(i, j)$ to $\beta'_{ijqN}(i, j)$. During the first scan, the entire MSB bits are marked and selected as significant. From the next scan onwards, the previous marked significant bits are considered to decide, whether the particular bit should be considered as significant or non-significant because all the bit planes are mutually dependent on each other. In order to select the significant bits in the second bit plane onwards, all the neighbour bit planes in the position $(i-1, j)$, $(i+1, j-1)$, $(i, j-1)$ and $(i-1, j-1)$ are considered into account. During the scanning process, if the selected bit is 1, then it is modeled as significant, and, if the selected bit is 0, then, bits in the above said neighbouring positions are checked, and if one of the neighbouring positions is 1, then the particular bit is also marked and selected as significant, otherwise, the bit is discarded. The arrangement of neighbouring positions is shown in Figure 2.
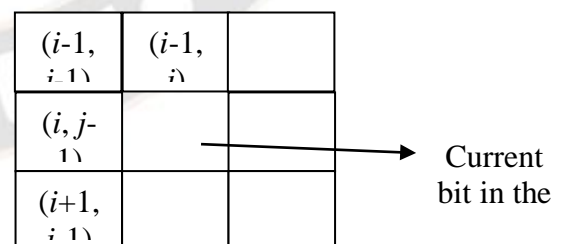

Figure 3. Arrangement of Neighbouring Bits towards Selection of Significant Bit

In the proposed orthogonal polynomials based multiresolution analysis image coding, the significant bits in each bit plane is modeled to form a sequence of contexts and geometrically distributed. The distribution is modeled as given Equation (9). For a context *c*, the data is modeled in the geometric probability distribution as:

$$\Pr(x = c) = (1 - p)^k p$$

(9)

2591

_____

where, p is the probability of the sequence of the bits in the bit plane and k is the number of runs of a bit. The modeled context is stored in priority queue based on the probability of the context. Each context is then subjected to the entropy coding with the modified Golomb-Rice encoder as discussed in the next sub section. Due to mutual dependence of the entire bit plane, the probability of the one bit plane is jointly distributed with the neighbour bit plane's data $\beta'_{ijq(i+1,j-1)}(x,y)$, $\beta'_{ijq(i,j-1)}(x,y)$, $\beta'_{ijq(i-1,j-1)}(x,y)$ and $\beta'_{ijq(i-1,j)}(x,y)$ for the particular modeled context. So, the overall real probability of the bit is quite different, which is significantly affected by the neighbourhood coefficients.

Therefore, the proposed orthogonal polynomials based multiresolution analysis image coding models the data using joint probability distribution for a context in terms of current bit plane data with their previous and next bit planes for selection of significant bits. For N bit planes (B_1, B_2, …, B_N) in a context c, the joint probability function will be:

P (b_1, b_2, …, b_N) = P($B_1 = b_1$, $B_2 = b_2$, …, $B_N = b_N$)   (10)

The steps involved in the selection of significant bits are given as an algorithm hereunder:

Algorithm: Selection of significant bits with joint probability bit modeling

Input: Bit plane of scalar quantized orthogonal polynomials transformed coefficients

Output: Selection of significant bit

Begin

Step 1: Decompose the quantized orthogonal polynomials coefficients in to N number of bit planes.

Step 2: Perform column wise scanning from $\beta'_{ijqN}(i,j)$ to $\beta'_{ijqN}(i,j)$.

Step 3: Mark and select all the MSB bits .

Step 4: To select next bit, consider the following positions $(i+1, j)$, $(i+1, j-1)$, $(i, j-1)$ and $(i-1, j-1)$.

Step 5: If the next bit is 1, it is marked and selected as significant.

Step 6: If the next bit is 0, check the positions in step 4, if any one the position is 1, consider as significant, else, discard the bit.

Step 7: Repeat the above process for all the sub bands.

End

After the selection of significant bits, the proposed orthogonal polynomials based multiresolution analysis image coding scheme performs entropy coding on the significant bits to produce the compressed data and same is presented in the next sub section.

F.   Modified Golomb-Rice Entropy Coding

For each bit plane the data is a single bit stream of length n bits in which, there are r 1s and rest of the bits are 0s. The proposed orthogonal polynomials based multiresolution analysis image coding makes a sequence of contexts that always ends with a 1 or 0 based on the most probable bits (MPB) i.e. $0^i 1$ and $1^i 0$. So, it is considered that each run is, to be of length L_1, $L_2,$ ....,L_r respectively. In the proposed orthogonal polynomials based multiresolution analysis image coding, modified Golomb-Rice encoder function takes as input the sequence of contexts together with their respective probabilities and encode accordingly. Each context is in the form of Geometric distribution, the number of runs i of the most probable bit is considered in the form $i=qm + r$, where q represents the quotient, r represents the remainder and m is a positive integer. Since, the value of m affects the overall compression rate, in the proposed orthogonal polynomials based multiresolution analysis image coding, optimal value of m is modeled as nearest power of 2 of $p*Ln(2)/(1-p)$ where, p is the probability of MPB in the modeled context. Finally, the q and r values are encoded in unary and binary coding in the form of $0^i 1$ as $1^q 0 y$ and $1^i 0$ as $0^q 1 y$ to produce the compressed data. The steps involved in the proposed entropy coding are given as an algorithm hereunder:

Algorithm : Modified Golomb-Rice Entropy Coding

Input :   Significant bits from all sub bands

Output : Compressed (encoded) data

Begin

Step 1:   Read the context and check for the last bit.

Step 1.1: If last bit is 1 then 0 is the MPB in the context.

Step 1.2: If last bit is 0 then 1 is the MPB in the context.

Step 2:   Find the probability p of the MPB in the particular context.

Step 3: Calculate the value of $p*Ln(2)/(1-p)$ and let m be the nearest power of 2 of $p*Ln(2)/(1-p)$.

Step 4:   Calculate the value of $log2\ m$.

Step 5:   Code each $0^i 1$ as $1^q 0 y$ and $1^i 0$ as $0^q 1 y$, where $i=qm+r$; $0 <= r < m$ and y is the binary coding of r, using $log2\ m$ bits.

Step 6:   Get final coded bit stream : MPB code1 code2 ... code n tail bit.

 End

The steps involved in the modified Golomb-Rice decoding procedure in the reconstruction of decompressed image in the proposed orthogonal polynomials based multiresolution analysis image coding are given as an algorithm hereunder:

Algorithm: Modified Golomb-Rice decoding

Input: Compressed (encoded) data

Output: Significant bits

Begin

Step 1: Read first bit and store it as MPB of the encoded stream.

Step 2: If MPB is 0 then calculate the number of occurrence of 1 till the next hit of 0 as q

Step 3: If MPB is 1 then calculate the number of occurrence of 0 till the next hit of 1 as   q

Step 4: Escape code with 0/1

Step 5: Read the next log2 m number of bits and store equivalent decimal as r

Step 6: Repeat the above steps for all code streams till the tail bit

Step 7: Calculate the original sequence of $0^i 1$ (or) $1^i 0$ using $i = qm+r$ $(0 <= r < m)$

Step 8: Arrange the final decoded bit streams for the each context

End

_____

G.   *Proposed Coding Algorithm*

The input for the proposed compression algorithm is uncompressed original image, *I* of size $(M \times N)$. At first, the proposed coding technique divides the original image into $(m \times n)$ non-overlapping bocks and orthogonal polynomials transformation as described in the section 3 is applied to each divided blocks. Then, the obtained transformed coefficients $\beta_{ij}^{'}$ are reordered into sub bands like structure and as described in the sub section 4.2.1 followed by a scalar quantization as described in the sub section 4.2.2. The quantized coefficients in the sub bands are converted to bit stream and represented in bit plane format. Then, the proposed orthogonal polynomials transformation based image compression algorithm performs selection of significant bits in the bit planes as described in the sub section 4.2.3. Then, the geometrically modelled bits are sent to entropy coder to produce yields a compressed data as described in the sub section 4.2.4. The steps involved in the proposed orthogonal polynominals based multiresolution analysis image compression technique are given as an algorithm hereunder:

Algorithm: Complete algorithm of proposed orthogonal polynomial based multiresolution analysis image coding

  Input   : Original Image, *I* of size $(M \times N)$, block size n.

Output   : Compressed image

Begin

Step 1: Read the Input Image, *I*.

Step 2: Divide the input image into $(m \times n)$ non-overlapping blocks, where $n << M, N$.

Step 3: Apply forward orthogonal polynomials transformation to divided blocks, as described in section 3.

Step 4: Represent the transformed coefficients $\beta_{ij}^{'}$ into sub bands like structure, as described in the section 4.2.1.

Step 5: Apply scalar quantization to entire transformed coefficients in the all sub bands, as described in the section 4.2.2.

Step 6: Convert the quantized coefficients into bits and represent it as bit plane format.

Step 7: Do Selection of significant bits as described in the sub section 4.2.3.

Step 8: Arrange all the bits into 1-D format.

Step 9: Arrange the bits into $p^n q$ format.

Step 10: Perform modified Golomb Rice entropy coding, as described in the section 4.2.4

Step 11: Calculate most probable bit from total bits.

Step 12: Calculate *q* & *r* for each group with *p*.

Step 13: Represent *q* as unary coding and *r* as binary coding

Step 14: Arrange *q* & *r* bits as encode bits.

Step 15: Synthesize the block and obtain compressed image, I_c.

End

IV. Performance Measures

To evaluate the effectiveness of the proposed orthogonal polynomials based multiresolution analysis image compression, standard measure viz. Peak signal to noise (PSNR).

Mathematical formula for computation of PSNR is:

$$PSNR = 10 \log_{10} \left[\frac{255^2}{\sqrt{MSE}} \right] \qquad (11)$$

where the Mean Square Error, MSE is given as,

$$MSE = \frac{1}{m \times n} \sum_{i=1}^{m} \sum_{j=1}^{n} \left[I(i,j) - I^{'}(i,j) \right]^2 \qquad (12)$$

$I_1(i,j)$ and $I_2(i,j)$ represent the intensity of the original image and decompressed image pixel positions at (i,j), and *m* and *n* are the height and width of the image.

V.   Experiments And Results Analysis

The proposed orthogonal polynomials based multiresolution analysis image compression has been experimented with more than 200 grey scale images of different types. Two sample images viz. Lena and Boat of size (256×256) with pixel values in the range of 0-255 are presented in figure 4(a) and 4(b) respectively .

(a)   (b)
Figure 4. Original Images (a) Lena (b) Boat

The original input images are divided into non-overlapping blocks of size (8×8) and each block is subjected to orthogonal polynomials transformation as described in the section 3 and reordered to subband like multiresolution structure as described in the section 4.2.1 . Then, scalar quantization is performed to reduce the precision as described in the section 4.2.2 and the proposed image compression algorithm considers bits per pixel (bpp) for quantization step which can be adaptively varied as per user's choice, and converts the quantized coefficients into bit streams. After, the quantized coefficients are converted to bit modeling in each bit plane, the proposed image compression algorithm performs selection of significant bits in the bit plane based on joint probability based scanning as described in the section 4.2.3 to form a context for entropy encoding as described in the section 4.2 4. The decompression procedure is just inverse of image compression process. The results of proposed orthogonal polynomials based multiresolution image compression on the input images shown in Fig. 4 are presented in Fig. 5 correspondingly, when bits per pixels in 0.25.

2593

_____

| (a) | (b) |

Figure 5. Results of proposed multiresolution analysis image compression with orthogonal polynomials (a) Decompressed Lena Image at 0.25 bpp (b) Decompressed Boat Image at 0.25 bpp

The performance of proposed orthogonal polynomials based multiresolution analysis image compression technique is measured in terms of PSNR, as described in section 5.For better understanding; the results of Lena and Boat image are also incorporated in the Table 1.

Table 1. Performance Measures of orthogonal polynomials based multiresolution analysis Image

Image	bpp	PSNR	ET (sec)
Lena	0.25	27.05	4.591
Girl	0.25	30.170	4.513
Barbara	0.25	22.119	3.995
Peppers	0.25	27.635	4.215
Cameraman	0.25	24.799	4.282
Boat	0.25	25.392	4.697

Table 2. Results of PSNR of Comparison of Proposed Orthogonal Polynomials Based Multiresolution Analysis with joint probability contact modelling, JPEG2000 and JPEG_LS Algorithm

Image	Bpp	Proposed Multiresolution Based Analysis	JPEG-LS (Zhen Liu)	JPEG2000 (Arandiga)
Lena	0.1	25.27	23.21	22.48
Lena	0.2	28.01	24.21	27.03
Lena	0.3	28.97	27.11	28.32
Lena	0.4	29.18	29.12	28.98
Boat	0.1	26.46	24.11	23.19
Boat	0.2	30.13	27.12	25.58
Boat	0.3	30.46	28.23	26.07
Boat	0.4	30.87	28.97	27.21

In order to measure the efficiency of the proposed orthogonal polynomials based multiresolution analysis image compression, experiments are conducted with JPEG2000 and JPEG-LS standard with different quality factors on same Lena image and boat image. The PSNR comparison results are presented in the Table 2 at varying bpp along with proposed orthogonal polynomials based multiresolution analysis **with joint probability contact modelling and modified** Golomb-Rice entropy coding. It is observed from the Table 2, the proposed orthogonal polynomials based multiresolution analysis image compression technique is better than JPEG2000 and JPEG-LS algorithm.

V. Conclusion

In this paper, orthogonal polynomials transformation with multiresolution approach has been utilized to propose an image compression algorithm. First, the orthogonal polynomials transformed coefficients are arranged in sub bands like structure and subjected to scalar quantization and bit plane representation. The proposed compression work selects significant bits from the bit plane with MSB and joint probability statistical model. The selected significant bits are encoded with modified Golomb-Rice coding to produce the compressed image. Experiments and analysis shows the efficiency of the proposed orthogonal polynomials based multiresolution analysis image compression in terms of PSNR.

References
1. Jain A.K, "Image Data Compression: A Review," Proceedings of IEEE, 69(3), pp. 349-389, 1981.
2. Gregory K Wallace, "The JPEG still picture compression standard," Communications of the ACM, Vol. 34, No. 4, pp. 30-44, 1991.
3. C. Christopoulos, "The JPEG2000 still image coding system : An overview," IEEE Transactions on Consumer Electronics, Vol. 46, No. 4, pp. 1103-1127, 2000
4. Ashin R, Morimoto A, Valliancourt R, "Image Compression with Multiresolution Singular Value Decomposition and other methods," Mathematical and Computer Modeling, Vol. 41, No.6-7, pp. 773-790, 2005.
5. Francesc Arandiga, Pep Mulet, Vicent Renau, "Lossless and near-lossless image compression based on multiresolution analysis," Journal of Computational and Applied Mathematics, Vol. 242, pp. 70-81, 2012.
6. Brendan J. Babb, Frank W. Moore and Pat Marshall, "Evolved multiresolution analysis transform for improved image compression and reconstruction under quantization," IEEE Symposium on Computational Intelligence in Image and Signal Processing, 2007 (CIISP 2007), pp. 202-207, 2007
7. M.N. Do and M. Vetterli, "The contourlet transform : An Efficient Directional Multiresolution Image Representation," IEEE Transactions on Image Processing, Vol. 14, No. 12, pp. 2091-2097, 2005.

2594

_____

8. John Michael Lounshery, "Multiresolution analysis for surface of arbitrary topological type," PhD Thesis, University of Washington, 1994.

9. Wei Hong, John Wright, Kun Huang and Vi Ma, "Multiscale hybrid linear models for lossy image representation," IEEE Transactions on Image Processing, Vol. 15, No. 12, pp. 3655-3671, 2006.

10. Candes E and Donoho DL, "New Tight frames of curvelets and optimal representation of objects with smooth singularities ," Technical Report, Standford University, CA, 2002.

11. Rakesh, Ganya, M. D. Babu, and G. Manoj Someswar. "A Novel Integrated Attestation Graph Analysis Scheme for Enhancing Result Quality and Higher Attacker Pinpointing Accuracy." International Journal of Research 3.2 (2016): 214-225.

12. Yair Wiseman, "The still image lossy compression standard – JPEG and Enhancement of JPEG Compression for GPS Images", *International Journal of Multimedia and Ubiquitous Engineering*, Vol.10, No.7, pp. 255-264, 2015.

13. Sandhya Vani, M., Durga Devi, R., Mandru, D.B. (2023). An Efficient Intrusion Detection Framework in Software-Defined Networking for Cyber Security Applications. In: Reddy, V.S., Prasad, V.K., Wang, J., Reddy, K.T.V. (eds) Soft Computing and Signal Processing. ICSCSP 2022. Smart Innovation, Systems and Technologies, vol 313. Springer, Singapore. https://doi.org/10.1007/978-981-19-8669-7_40

14. Rong Zhang, Rongshan Yu, Qibin Sun and Wai-Choong Wong, "A New Bit-Plane Entropy Coder for Scalable Image Coding," *IEEE International Conference on Multimedia and Expo*. pp. 237-240, 2005.

15. Minsoo Rhu and In-Cheol Park, "Optimization Of Arithmetic Coding For JPEG2000," *Circuits And Systems For Video Technology*, Vol.20, No.3, pp. 32-45, 2010.

16. Hong-Sik Kim, Joohong Lee, Hyunjin Kim, Sungho Kang, and Woo Chan Park, "A Lossless Colour Image Compression Architecture Using a Parallel Golomb-Rice Hardware CODEC," *IEEE Transactions on Circuits and Systems for Video Technology,* Vol. 21, No. 11, pp 1581 – 1587, 2011

17. Chang-Hoon Son, Ji-Won Kim, Sung-Gun Song, Seong-Mo Parklow and Young Min Kim, "Low complexity embedded compression algorithm for reduction of memory size and bandwidth requirements in the JPEG2000 encoder," *IEEE Transactions On Consumer Electronics*, Vol.56, No. 56, pp. 2421-2429, 2010.

18. Salah Ameer and Otman Basir, "Image compression using Plane fitting and inter-block prediction," *Image and Vision Computing,* Vol. 27, No. 4, pp. 385–390, 2009.

19. Mandru, Deena Babu, and Y. S. Krishna. "Multi-view Cluster Approach to Explore Multi-Objective Attributes based on Similarity Measure for High

Dimensional Data." International Journal of Applied Engineering Research 13.15 (2018): 12289-12295.

20. C Chrysafis and A. Ortega, "Efficient context-based entropy coding for lossy wavelet image compression," *IEEE Data Compression Conference*, pp. 241–250, 1997.

21. Zhen Liu and Lina J. Karam, "Mutual Information-Based Analysis Of JPEG2000 Contexts", *IEEE Transactions on Image Processing*, Vol. 14, No.4, pp.156-164, 2005.

22. D. Taubman, "High performance scalable image compression with EBCOT," *IEEE Transaction on Image Processing*, Vol. 9, No. 7, pp. 1158–1170, 2000.

23. Pooneh Bagheri Zadeh, Akbar Sheikh Akbari, Tom Buggy and John Soraghan, "Multiresolution HSV and statistically based image coding scheme," Multimedia Tools and Applications, Vol. 49, No. 2, pp. 347-370, 2010.

24. Henrique S. Malvar, "Adaptive Run-Length / Golomb-Rice Encoding Of Quantized Generalized Gaussian Sources with Unknown Statistics", *Data Compression Conference*, Vol. 6, pp. 23-32, 2000.

25. Jianhua Chen, "Context Modeling Based on Context Quantization with Application in Wavelet Image Coding," *IEEE Transactions on Image Processing*, Vol. 13, No. 1, pp. 26-32, 2004.

26. L. Ganesan and P. Bhattarcharyya, "Edge detection in un textured and textured images-a common computational framework," *IEEE Transactions on Systems, Man, and Cybernetics Society, Part B*, Vol. 27, No. 5, pp. 823-834, 1997.

27. R. Krishnamoorthy, "Transform coding of monochrome images with a statistical design of experiments approach to separate noise," *Pattern Recognit. Lett.*, vol. 28, no. 7, pp. 771–777, 2007.

28. R. Krishnamoorthy and N. Kannan, "A new integer image coding technique based on orthogonal polynomials," *Image Vis. Comput.*, vol. 27, pp. 999–1006, 2009.

29. Mandru, D.B., Aruna Safali, M., Raghavendra Sai, N., Sai Chaitanya Kumar, G. (2022). Assessing Deep Neural Network and Shallow for Network Intrusion Detection Systems in Cyber Security. In: Smys, S., Bestak, R., Palanisamy, R., Kotuliak, I. (eds) Computer Networks and Inventive Communication Technologies . Lecture Notes on Data Engineering and Communications Technologies, vol 75. Springer, Singapore. https://doi.org/10.1007/978-981-16-3728-5_52

30. R. Krishnamoorthy and J. Kalpana, "Fast Retrieval of Color Objects with Multidimensional Orthogonal Polynomials," *Multidimens. Syst. Signal Process.*, vol. 25, no. 4, 2013.

31. R. Krishnamoorthy and J. Kalpana, "Generalized Adaptive Bayesian Relevance Feedback in the Orthogonal Polynomials Domain," *Signal Processing*, vol. 92, no. 12, pp. 3062–3067, 2012.

32. J. Kalpana and R. Krishnamoorthy, "Color image

_____

retrieval technique with local features based on orthogonal polynomials model and SIFT," *Multimed. Tools Appl.*, vol. 75, no. 1, pp. 49–69, 2016.

33. Seshadri Ramana, K., Bala Chowdappa, K., Obulesu, O.,Mandru, D. B., & Kallam, S. (2022). Deep convolution neural networks learned image classification for early cancer detection using lightweight. Soft Computing, 26(12), 5937-5943.

34. R. Krishnamoorthy and S. Sathiyadevi, "A multiresolution approach for rotation invariant texture image retrieval with orthogonal polynomials model," *J. Vis. Commun. Image Represent.*, vol. 23, no. 1, pp. 18–30, 2012.

35. Mandru, Deena Babu. "A comparative study on Covid-19 cases in top 10 states/UTS of India in using machine learning models." Turkish Journal of Computer and Mathematics Education (TURCOMAT) 12.10 (2021): 4514-4524.

36. R. Krishnamoorthy and R. Punidha, "An edge preserving requantization model for color image coding with orthogonal polynomaials," *Digital Signal Processing*, Vol. 23, No. 4, pp. 1163-1170, 2013.

37. R. Krishnamoorthy and R. Punidha, "Low-bit rate multistage vector quantization based on energy clustered training set,"*Multimed. Tools Appl.*, Vol. 70 , no. 3, pp. 2293-2308, 2014.

38. R. Krishnamoorthy and G. Devasena, "A Systematic Analysis of Transform Coefficients and Block Decomposition for Texture Enhancement with Orthogonal Polynomials Model," *IEEE Transactions on Systems, Man and Cybernetics: Systems*, No. 99, pp. 1-9, 2016. DOI. 10.1109/TSMC.2016.2572305

39. Ranjan K. Senapati, Umesh C. Pati, Kamala K. Mahapatra, "Image Compression Using Discrete Tchebichef Transform Algorithm," International Conference on Advances in Communication, Network and Computing (CNC-2010), 4-5th October, Calicut, 2010

40. Krishna, Paruchuri Jeevan, M. Deena Babu, and G. Manoj Someswar. "Design & Development of an improvised PACK System using TRE Technique for Cloud Computing Users." International Journal of Research 3.2 (2016): 384-393.

41. Paulo A. M. Oliveira, Renato J. Cintra, Fabio M. Bayer, Sunera Kulasekera, and Arjuna Madanayake, "A Discrete Tchebichef Transform Approximation for Image and Video Coding," *IEEE Signal Processing Letters*, Vol. 22, No. 8, pp. 1137-1141, 2015.

42. Alexandre Krivoulets, "Efficient Entropy Coding for Image Compression," IT University Technical Report Series, 2002

43. Mandru, YK Sundara Krishna Deena Babu. "'Enhanced Feature Selection Clustering Algorithm for Attribute Similarity in High Dimensional Data', 2018." International Journal of Engineering & Technology 7: 688-693.

44. Seyun Kim and Nam Ik Cho, "Hierarchical Prediction and Context Adaptive Coding for Lossless Color Image Compression,"*IEEE Transactions on Image Processing*, Vol. 23, No. 1, pp. 445-449, 2014.

45. En-hui Yang and Longji Wang, "Joint optimization of Run-length coding, Huffman coding and Quantization table with complete baseline JPEG compatability", *IEEE International Conference on Image Processing*, pp. 181-184, 2007. DOI. 10.1109/ICIP.2007.4379276

46. En-hui Yang and Longji Wang, "Joint Optimization of Run-Length Coding, Huffman Coding, and Quantization Table With Complete Baseline JPEG Decoder Compatibility," IEEE Transactions on Image Processing, Vol. 18, No. 1, pp. 63-74, 2008.

47. Bruno Aiazi, Stefano Baronti and Luciano Alparone, "Lossless image compression based on an enhanced fuzzy regression prediction," International Conference on Image Processing (ICIP 99), Vol. 1, pp. 435-439, 1999.

48. Ramji Venkataramanan, Tuhin Sarkar, and Sekhar Tatikonda, "Lossy Compression via Sparse Linear Regression: Computationally Efficient Encoding and Decoding," IEEE Transactions on Information Theory, Vol. 60, No. 6, pp. 3265-3278, 2014.

49. Ranjan Kumar Senapati, Umesh C. Pati, Kamala Kanta Mahapatra, "Reduced memory, low complexity embedded image compression algorithm using hierarchical listless discrete Tchebichef transform," *IET Image Processing*, Vol. 8, No. 4, pp. 213-238, 2014.

50. R. Krishnamoorthy and P. Bhattacharya, "Color edge extraction using orthogonal polynomials based zero crossings scheme," Information Sciences, Vol. 112, No. 1-4, pp. 51-65, 1998.

51. R. Krishnamoorthy and P. Murali, "A selective image encryption based on square-wave shuffling with orthogonal polynomials transformation suitable for mobile devices," Multimedia Tools and Applications, 2015.

52. Yamuna Devi, M.M., Jeyabharathi, J., Kirubakaran, S. et al. Efficient segmentation and classification of the lung carcinoma via deep learning. Multimed Tools Appl (2023). https://doi.org/10.1007/s11042-023-17082-2.

53. Madhusekhar, Y., Sandhya Priyanka, P., Mandru, D.B., Srikanth, T. (2023). Blockchain: A Safe Way to Transfer Signatures in a Distributed Intrusion Detection System. In: Manchuri, A.R., Marla, D., Rao, V.V. (eds) Intelligent Manufacturing and Energy Sustainability. Smart Innovation, Systems and Technologies, vol 334. Springer, Singapore. https://doi.org/10.1007/978-981-19-8497-6_26.

54. T.Srikanth, et al. "Commonly Used Algorithms in Data Science Along with Internal Logics and Implementations Through R Programming". International Journal of Engineering and Advanced Technology (IJEAT), vol. 9, no. 3, Zenodo, Feb. 2020, pp. 2649–52, doi:10.35940/ijeat.C5811.029320.